**Projectile Motion (Summary)**

- A projectile is an object in free fall: subject to gravity and air resistance
- In this context, we will ignore the effects of air resistance; sometimes, this is not a good idea
- When air resistance is ignored, projectiles are a good example of constant acceleration
- Projectile position, displacement, velocity, and acceleration can be better understood using the equations of constant acceleration

**Galileo’s equations of constant acceleration:**

\[
\begin{align*}
  v_2 &= v_1 + at \\
  d &= v_1 t + \frac{1}{2}at^2 \\
  v_t^2 &= v_i^2 + 2ad
\end{align*}
\]

- \(d\) = displacement
- \(v\) = velocity
- \(a\) = acceleration
- \(t\) = time
- \(v_i\) = initial time
- \(v_f\) = final time

The purpose of these equations is to help us estimate instantaneous position, displacement, velocity, and acceleration for projectiles.

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**Projectile Motion**

With no air resistance, the path followed by a projectile will be a **parabola**

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**Vertical Velocity (\(V_v\))**

Horizontal and vertical components of projectile motion should be treated separately
Gravity will cause the vertical component of velocity to change during the flight

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**Horizontal Velocity (\(V_h\))**

Horizontal velocity is constant throughout flight. Why?

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**Projectile Motion**

In review, if air resistance is negligible:
- Horizontal velocity (\(V_h\)) is constant
- Vertical velocity (\(V_v\)) changes by -9.81 m/s/s

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**Velocity Diagram**

[Diagram showing velocity components]
Vertical Motion of a Projectile

The vertical motion of a water balloon, dropped from the edge of a building, is very predictable.

Hint, use: \[d = v_i t + \frac{1}{2} a t^2\]
and \[v_f = v_i + a t\]

<table>
<thead>
<tr>
<th>time</th>
<th>position</th>
<th>velocity</th>
<th>acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 s</td>
<td>0 m</td>
<td>0 m/s</td>
<td>-9.8 m/s²</td>
</tr>
<tr>
<td>1 s</td>
<td>4.9 m</td>
<td>-9.8 m/s</td>
<td>-9.8 m/s²</td>
</tr>
<tr>
<td>2 s</td>
<td>-19.6 m</td>
<td>-19.6 m/s</td>
<td>-9.8 m/s²</td>
</tr>
<tr>
<td>3 s</td>
<td>-44.1 m</td>
<td>-29.4 m/s</td>
<td>-9.8 m/s²</td>
</tr>
</tbody>
</table>

Another example

How high was the cliff (what is \(d_H\))? 

Horizontal displacement \(d_H\) or range of a projectile is the main index of performance in many cases of projectile motion.

If air resistance is negligible, there is no net force in the horizontal direction \((\Sigma F_H = 0; a_H = 0)\)

Given the equation: \[d = v_H t + \frac{1}{2} a t^2,\]
we assume that: \(d_H = V_H \times t\)

What about horizontal motion?

Horizontal Displacement

\(d_H = V_H \times t\), indicates that horizontal displacement depends on horizontal velocity and flight time. What affects horizontal velocity?
Time of Flight
What affects flight time? (two primary factors)
1. Vertical speed at release, which is affected by angle of release
2. Height of release

Some Application
Based upon what we have learned, what factors can one manipulate to influence the horizontal displacement of a projectile?

Relative Height of Release
Speed of Release
Angle of Release

Some Application
Does increasing the height of release always lead to greater \( d_H \)?
Yes, why?
\[ d_H = v_h \times t \]

Does increasing speed of release always lead to greater \( d_H \)?
Yes, but again, why?
\[ d_H = v_h \times t \]

Some Application
To which of the three aforementioned factors (height of release, speed of release, and angle of release) is \( d_H \) most sensitive?
\[ d_H = v_h \times t \]

Speed of release can positively affect \( v_h \) and \( t_{\text{TOT}} \)

What are the performance-related implications of this information?

More Application...
How do actual angles of release in various sports compare to the theoretical optimum?

Shot Put:
- Positive height of release, so optimal angle should be slightly lower than 45°
- Theoretically optimal angle is about 40-41°
- Skilled shot-putters use angles of 35-37°
- Close, but why the discrepancy?
More Application…

Long Jump:
Positive height of release, so optimal angle should be slightly lower than 45°

Projectile Motion: Theory vs Reality

Long Jump:
- Theoretically optimal angle is about 42°
- Top long jumpers use angles of 17-23°
- Very different. Why the major discrepancy?

When traveling at ~10 m/s, there is not enough time to generate a large takeoff angle
Long jumpers sacrifice optimal angle to get maximum speed.